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A new estimation method of thermal diffusivity using analytical inverse solution for one-dimensional heat conduction

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Abstract

A new procedure to estimate thermal diffusivity has been developed by using a solution explicitly obtained from an inverse problem of one-dimensional unsteady heat conduction. This method has merit being independent of surface condition and is simpler than the existing methods, which commonly use a direct heat conduction solution and are strongly subject to a boundary condition. The thermal diffusivity can be estimated by using a change in the temperatures, which includes some errors at two different points as uncertainty in the measurement. The value estimated thereby is found to be in good agreement with that of the tested materials, while thermal conductivity needs a reference to give a known heat to the tested material. This method makes the simultaneous measurement of both thermal diffusivity and thermal conductivity possible by using a reference material.

1. Introduction

A problem to estimate a thermal property from a temperature change measured in a solid is considered as one of the inverse problem. However, most of the methods to measure thermal properties such as thermal diffusivity, heat capacity and thermal diffusivity usually employ a direct solution for one-dimensional transient heat conduction and determine the thermal properties as to fit the measured temperature change to the corresponding direct solution. As for the usage of the direct solution, so far, several methods [1–3] have been developed for different heating procedures such as (1) pulsating heating [4–6], (2) stepwise heating [6], (3) periodic heating [7] and (4) continuous heating [8]. The accuracy of the values estimated thereby is strongly influenced by a difference between actual and conceptual boundary conditions and by measured accuracy due to the application of the corresponding direct solution for the conceptual condition. Therefore, a special technique may be needed to sustain a boundary condition and in the measurement of the temperature.

Apart from these methods, a different procedure being independent of boundary condition was proposed by Iida and Shigeta [9,10], in which instead of using the direct solution, the thermal properties are determined by using subsidiary solution obtained by Laplace transformation. This is called arbitrary heating method, which is independent of the heating method.

On the other hand, Alifanov [11] briefly described a method to estimate the thermal properties using an inverse solution. Provided that the inverse solution can predict the boundary condition with a satisfactory accuracy, the value of the properties estimated thereby becomes free from the boundary condition and then would be measured easier than that using the direct solution.

Recently, Monde [12] has derived an explicit solution for the inverse problem for one-dimensional transient heat conduction using Laplace transformation and shows that the solution can predict the corresponding

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Nomenclature

a	thermal diffusivity
a_{pre}	estimated value of thermal diffusivity
a_{true}	actual value of thermal diffusivity
$f_i(t)$	function of non-dimensional temperature at a point of x_i
k	rate of temperature rise
L	characteristic length
$\min(T/T_0)$	minimum of significant number or division of measuring equipment
N	degree of approximate polynomial
q	heat flux
s	Laplace operator $(= ap^2)$
T	temperature
T_i	initial temperature
T_L	temperature at the point of x_1 at the time of $t/t_L = 1$
t	time
t_i^*	non-dimensional time lag $(\mathrm{erfc}(x_i/2\sqrt{at_i^*}) = \min(T/T_0))$ at a position of x_i
x	x-coordinate
ε	normal random value of [−1,1]
λ	thermal conductivity
$\Phi(\xi,\tau)$	non-dimensional heat flux
$\bar{\theta}(\xi,s)$	subsidiary value of θ
$\bar{\Phi}(\xi,s)$	subsidiary value of Φ
ξ	non-dimensional distance $(= x/L,\ \xi_1 < \xi_2)$
τ	non-dimensional time $(= at/L^2)$

surface temperature and heat flux well and then is also robust for a disturbance included in the measured values. The estimated surface temperature and heat flux can be predicted with the same level as the measured temperature.

A new method is proposed to estimate the thermal diffusivity using the inverse solution and its applicability will be verified by the thermal diffusivity. This method has merit in that these values can be obtained independent of surface condition. This method is shown to be easily extended to the simultaneous measurement of thermal diffusivity and thermal conductivity using a reference material to give a known heat transfer to the tested material.

2. Inverse solution of one-dimensional heat conduction

Monde [12] has already given the inverse solution for one-dimensional heat conduction equation with constant properties in detail. Therefore, the inverse solution needed only to estimate thermal properties using a semi-infinite body is briefly described here, since the solution for the semi-infinite body becomes simpler than that for a finite body and is well within a measured time, which will be discussed later.

One-dimensional heat conduction equation with constant thermal property can be written as

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}. \tag{1}$$

A subsidiary form after Laplace transformation can be expressed for an initial condition of $T = 0$ as

$$\frac{d^2\overline{T}}{dx^2} - p^2\overline{T} = 0. \tag{2}$$

Let us set the initial condition $T = 0$ for a constant initial temperature so that it does not lose any generality.

The general solution of Eq. (2) can be easily derived as

$$\overline{T}(x,s) = Ae^{px} + Be^{-px}, \tag{3}$$

where $p^2 = s/a$, s the Laplace's operator and A and B are integral constants subject to a surface condition.

In order to determine the constants A and B, one needs the two temperatures measured at two different points $(x = x_1, x_2,\ x_2 > x_1 > 0)$ in a finite body. However, if a measurement of the temperature would be completed within the time given by $\mathrm{erfc}(x_2/2\sqrt{at}) = \min(T/T_0)$, then the part in the body being larger than the length of x_2 does not experience any heat and temperature change and then this body can be considered as the semi-infinite one. For this case, only one constant B should be determined from the temperature only at the one point, since the constant A immediately becomes 0. Let the temperature change at the point of $x = x_1$ be $T(x_1,t) = T_0 f_1(t)$. In addition, the temperature change can be approximated as

$$f_1(t) = \sum_{k=0}^{N} \frac{b_k}{\Gamma(k/2+1)}(t - t^*)^{k/2}, \tag{4}$$

where in Eq. (4) coefficients b_k can be determined, for example, by using the least-mean square method from the measured temperature, t^* is a time lag, which it takes for a temperature to be monitored at the measuring point and N gives a degree of half polynomial series of time. This time lag can be determined as $\mathrm{erfc}(x_1/2\sqrt{at^*}) = \min(T/T_0)$.

After transforming Eq.(4) in subsidiary form, one may substitute it into Eq. (3) giving the constant B and then the temperature in the body reaches as

$$\frac{\overline{T}(x,s)}{T_0} = e^{-p(x-x_1)}e^{-st^*}\sum_{k=0}^{N} b_k/s^{(k/2+1)}. \tag{5}$$

The surface temperature becomes in subsidiary form as follows:

$$\frac{\overline{T}(0,s)}{T_0} = e^{px_1} e^{-st^*} \sum_{k=0}^{N} b_k / s^{(k/2+1)}. \tag{6}$$

Consequently, the surface temperature can be obtained by executing Laplace's inverse transformation

$$\frac{T_w(t)}{T_0} = \sum_{j=-1}^{N} E_j (t - t^*)^{j/2} \Big/ \Gamma\left(\frac{j}{2} + 1\right), \tag{7}$$

where

$$E_{-1} = \sum_{k=0}^{Nk} b_k e_{k+1}, \qquad j = -1, \quad Nk = N,$$

$$E_j = \sum_{k=0}^{Nk} b_{k+j} e_k, \qquad j \geqslant 0, \quad Nk = N - j,$$

$$e_k = \frac{(x_n/\sqrt{a})^k}{k!}, \qquad n = 1, 2.$$

The details of how to derive these constants are depicted in [12]. In addition, it is mentioned in [12] that the inverse solution can predict the surface condition with the same accuracy as in measuring the temperatures in the body when the first derivative of $f_1(t)$ with respect to time becomes continuous except for $t = 0$.

3. Estimation of thermal diffusivity

For the case of constant thermal diffusivity, the surface temperature estimated from Eq. (7) using the temperature measured at another point x_2, theoretically becomes the same as that estimated from the measured temperature at the point x_1. Therefore, the thermal diffusivity can be determined such that two surface temperatures of $T_{w,1}$ and $T_{w,2}$ estimated from the points of x_1 and x_2 become equal over a measured time. That is, the thermal diffusivity can be determined as to satisfy the following equation:

$$F(a) = \int_{t_1}^{t_2} (T_{w,1}(t,a) - T_{w,2}(t,a))^2 \, dt \to 0, \tag{8}$$

where t_1 is a time larger than the minimum predictive time after which the surface temperature can be predicted with an accuracy of 99% compared with the corresponding exact one. It is understood that t_1 is usually determined by $at_1/L^2 = 0.01$ [12]. As for the time of t_2, it should be shorter than t_L determined for the tested body to satisfy the semi-infinite body, that is, $\mathrm{erfc}(L/2\sqrt{at_L}) = \min(T/T_0)$. This means $t_2 \leqslant t_L$. Incidentally, the thermal diffusivity is unknown, yet. Therefore, an approximate value should be first assumed to estimate the times of t_1 and t_L. The actual value is finally given by iteration as to satisfy Eq. (8).

3.1. Approximate equation and measuring temperature

As for the order of N in Eq. (4), $N = 5$ is employed here, since it is mentioned in [12] that $N = 5$–7 was proved enough for the surface temperature to be predicted by Eq. (7).

The present method is first checked by using the artificial data consisting of the exact values calculated from a direct solution and a value of uncertain measurements. When one measures a temperature using thermocouple, the level, N_{sf} of a significant digit for the temperature obtained may be two or three. Therefore, the uncertain measurements can be evaluated by the following two methods:

1. To cut off values calculated from an exact solution at a certain significant digit, namely

$$\frac{T(x_n,t)}{T_0} = \mathrm{Int}\left(\frac{T_{exact}(x_n,t)}{T_0} \times 10^{N_{sf}} + 0.5\right) \Big/ 10^{N_{sf}}, \quad n = 1, 2 \tag{9}$$

2. To superimpose a normal random value ranging from -1 to 1 and having a standard deviation, $\sigma = 1$

$$\frac{T(x_n,\tau)}{T_0} = \frac{T_{exact}(x_n,t)}{T_0} + 0.005\varepsilon(m = 0, \sigma = 1), \quad n = 1, 2. \tag{10}$$

Eq. (10) shows the case of $N_{sf} = 2$ as an example.

It is found in [12] that for the same significant digit, the values estimated by using Eq. (10) gave the worse predicted result than those by using Eq. (9). Here, Eq. (10) is employed to evaluate each coefficient of Eq. (4).

Substituting the surface temperatures estimated from the temperatures measured at two different points into Eq. (8), one executes integration, resulting in a function of thermal diffusivity only, namely $F(a)$. The value of a, which we have called as thermal diffusivity, can be determined such that the value of $F(a)$ becomes a minimum value, that is, $dF(a)/da = 0$. Incidentally, it may be worth discussing the characteristics of $dF(a)/da$. The function of $dF(a)/da$ becomes a very complicated form including many terms, for example, 28 terms for $N = 5$, related to the different forms of a, but the function of $dF(a)/da$ is found to show a monotonic increase from $-\infty$ to ∞ with increasing the value of a for $a > 0$. Therefore, it is very easy to numerically search the root of $dF(a)/da = 0$ within the range of $a > 0$.

It may be worth mentioning, finally that in the process of determining the thermal diffusivity, this method is totally independent of the surface condition, that gives an essential advantage compared with those using direct solution which may be strongly influenced by a difference between the assumed and actual surface conditions.

3.2. Concrete procedure to determine thermal diffusivity

The thermal diffusivity can be determined by the following steps:

1. The times of t_1 and t_L in Eq. (8) are estimated by first assuming appropriate thermal diffusivity.
2. The time lags of t_1^* and t_2^* at two measuring points of x_1 and x_2 are also estimated.
3. The temperatures at the points of x_1 and x_2 are given by Eq. (10).
4. The coefficients, b_k, in Eq. (4) are determined as to approximate the corresponding temperature.
5. The coefficients, E_j, in Eq. (7) are also calculated.
6. The thermal diffusivity is given by searching the root of $\mathrm{d}F(a)/\mathrm{d}a = 0$.
7. The iterative procedure will continue until the difference between the first assumed and the finally obtained thermal diffusivities becomes smaller than a prescribed value, usually being accurate within three significant digits.

As for the value of t_2, it is basically enough for t_2 to satisfy $t_2 < t_L$, and how the value of t_2 influences the accuracy of the prediction, will be discussed later.

4. Result and discussions

It is found from [12] that the inverse solution gives a better prediction for the surface condition in the case where the surface condition smoothly changes with time for $t > 0$.

Here, as an example case three different surface conditions will be dealt with for the initial temperature of $T = 0$.

1. The surface temperature is kept constant, namely $T_w/T_0 = 1$.
2. The surface temperature is proportionally increased with time, namely $T_w/T_0 = kt$.
3. The surface heat flux is kept a constant, namely $q_w/q_0 = 1$ (the surface temperature is given in a textbook, for example, see [13]).

One can estimate the thermal diffusivity using Eq. (10), where the exact solution for the surface condition is inserted and then by following the concrete procedure.

Fig. 1 shows a variation of $\mathrm{d}F(a)/\mathrm{d}a$ when a copper surface is subject to a boundary condition, case (3). It also shows the actual value of thermal diffusivity, a_{true}, for copper being $a_{\mathrm{pre}} = 1.18 \times 10^{-4}\ \mathrm{m^2/s}$.

It is found from Fig. 1 that the root of $\mathrm{d}F(a)/\mathrm{d}a = 0$ has only one value for $a > 0$. The value of this root just corresponds to the estimated value of a.

4.1. Measuring time and sampling number

The measuring time required to collect the temperature data is available within the time of t_L, namely

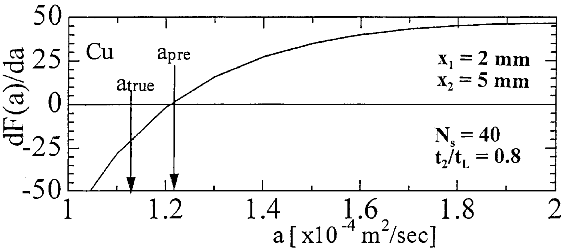

Fig. 1. Value of $\mathrm{d}F(a)/\mathrm{d}a$ vs a (case(3)).

$t_2 < t_L$. However, from an engineering point of view, a suitable measuring time should be given to avoid an unnecessarily long measuring time.

If an expected accuracy in the measurement was to become two significant digits, then $\min(T/T_0) = \mathrm{erfc}(L/2\sqrt{at_L}) = 0.01$ for which the maximum measuring time can be given by $L/2\sqrt{at_L} = 1.86$. The value of t_L/t_1 becomes $t_L/t_1 = 7.23$, that is, the measuring time becomes about seven times shorter than that of t_1.

Fig. 2 shows an effect of the measuring time and the sampling number of the data on accuracy in estimating

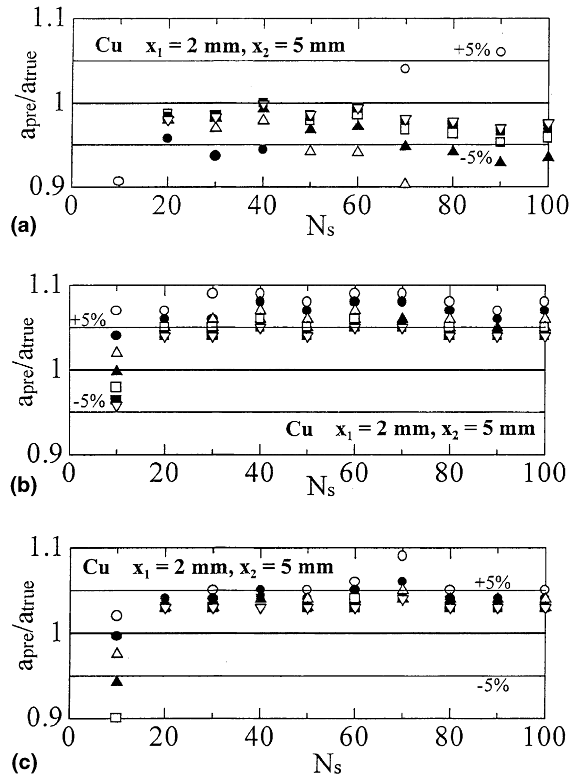

Fig. 2. Effect of sampling number on accuracy of estimated values (t_2/t_L: ○: 0.27, ●: 0.40, △: 0.53, ▲: 0.66, □: 0.80, ■: 0.93, ▽: 1.0). (a) step increase in temperature (case(1)); (b) linear increase in temperature (case(2)); (c) step increase in heat flux (case(3)).

the value of thermal diffusivity for three different cases of (1)–(3). The temperatures used for the estimation are obtained, for example, at the positions of $x_1 = 2$ mm and $x_2 = 5$ mm. It is worth mentioning that the same result in any material can be expected since the inverse solution is given in a non-dimensional form. Therefore, as an example, the predicted results for copper are shown in Fig. 2. Figs. 2(b) and (c) show that for the cases (2) and (3) the surface temperature gradually rises, the estimated values fall down within a similar accurate range of ±2–3% for the same value of t_2/t_L being independent of the sampling number and then the estimated values become worse with a decrease in the measuring time, t_2. On the other hand, Fig. 2(a) shows that in case (1) the surface temperature suddenly rises, the estimated value gradually becomes worse with an increase in the sampling number for shorter measuring time. For the measuring time of $0.66 \leqslant t_2/t_L \leqslant 1.0$, however, the estimated value converges within a certain accurate range of ±2–3% for the present sampling number of $N_s = 20$–100, while for $t_2/t_L \leqslant 0.53$, the estimated value does not converge with an increase in the sampling number. This result may be due to an increase in a relative error, since the absolute error of 0.005ϵ is added to the exact value of T/T_0 as the uncertain measurement. Under such conditions, the increase in the sampling number makes the relative error amplify compared with an exact value. In the cases (2) and (3), the estimated value also becomes worse compared to that of the desired result with decreasing t_2/t_L although it does not diverge in $t_2/t_L \leqslant 0.53$. This is also due to the same reason. The reason that the estimated value does not diverge is due to the continuity of the first derivative with respect to time by which the inverse solution was guaranteed to give at high accuracy. It is found from Fig. 2 that for the measuring time of $0.8 \leqslant t_2/t_L \leqslant 1.0$, the thermal diffusivity can be obtained at high accuracy at the sampling number of 20.

Incidentally, in order to calculate the maximum measuring time, t_L, one needs the value of the thermal diffusivity, which is unknown yet. After two or three trials of the measurement, however, an approximate value could be easily found.

Finally, the values predicted by this method gather around the true value and tend to deviate either larger than or smaller than the true value. Such characteristics come from the degree of approximate polynomial equation.

4.2. Effect of measuring points

Fig. 3 shows an effect of the distances of the measuring point, x_2, on accuracy of the estimated value under the condition of $t_2/t_L = 0.8$, $N_s = 40$, $x_1 = 2$ mm. The point of $x_1 = 2$ mm is selected as a point closer to the surface for two reasons. It is a realistic matter in an actual measurement and to obtain accuracy of the inverse solution, which is guaranteed by choosing the measuring point as close to the surface as possible. Fig. 3 shows that the accuracy of the estimated value gradually deteriorates as another measuring point, x_2, goes away from the surface. This result is mainly due to the characteristics of the inverse solution that its accuracy deteriorates with increasing the distance from the surface to the measuring point. Fig. 3 finally recommends us to choose the distance of x_2 to be less than 10 mm for any case, but not to be less than about 3 mm in actual measurement to avoid an influence of inserted thermocouple on heat flow in the solid. It may be not necessary to say that from a mathematical point of view, it is better for the point x_2 to be closer to the point x_1.

Fig. 4 shows the values estimated for several solid materials by applying this method for the case (3). The value of thermal diffusivity can be estimated within an accuracy of 2–3% in the simulation as shown in Fig 4.

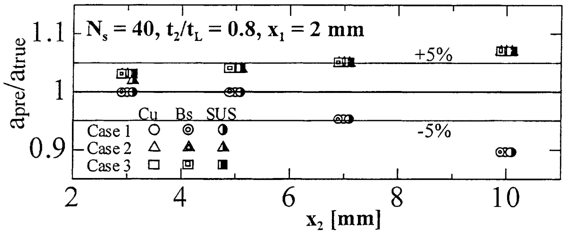

Fig. 3. Effect of position of measurement on accuracy of predicted values.

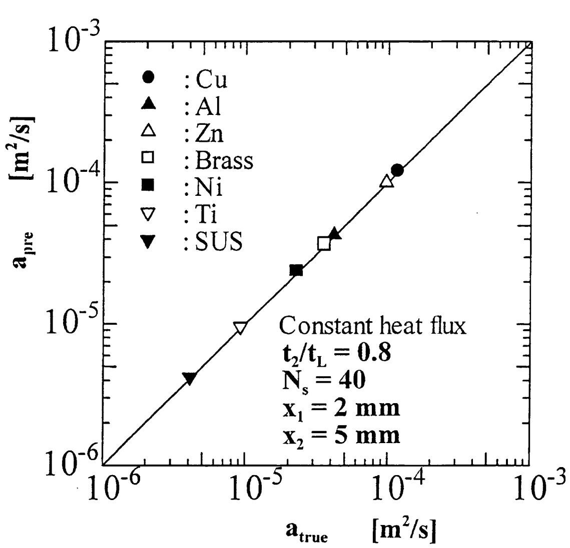

Fig. 4. Comparison of estimated and actual values (case(3): $t_2/t_L = 0.8$, $N_s = 40$).

4.3. Instruction and recommendation to actual measurement

In an actual measurement, the two times needed to evaluate Eq. (8) are recommended: for the time of t_1, the time is given by $at_1/L^2 = 0.01$ and for the time of t_2, $0.8 \leqslant t_2/t_L \leqslant 1.0$ and t_L is given by $\mathrm{erfc}(L/2\sqrt{at_L}) = \min(T/T_0)$. The measuring point of x_2 is recommended so as to satisfy $x_2/L < 0.2$ for the plate [12].

5. Experiment of measurement and validation

The procedure to numerically estimate thermal diffusivity using the inverse solution was explained in the previous sections. In this section, one will discuss the availability of the proposed method by actual measured temperatures at two different points and comparing the estimated thermal diffusivity with a measured one.

5.1. Experimental apparatus

Fig. 5 shows a schematic of an experimental apparatus and dimensions of the test material. A commercial handy plug air jet heater is used as a heat source to heat the test material of 100 mm in length. Two thermocouples are embodied into two different positions of 2 and 5 mm from the surface to measure their temperatures. The actual positions of the thermocouples are directly measured by removing the area between the two thermocouples. Minimum division of the temperature measuring equipment used is 0.06 K for copper and brass, while it is 0.12 K for stainless steel. The frequency response of the equipment is 20 Hz and then the shortest sampling time becomes 0.05 s.

5.2. Temperature change at the measuring point

Fig. 6 shows the temperature changes for copper and stainless steel plotted against non-dimensional time, t/t_L

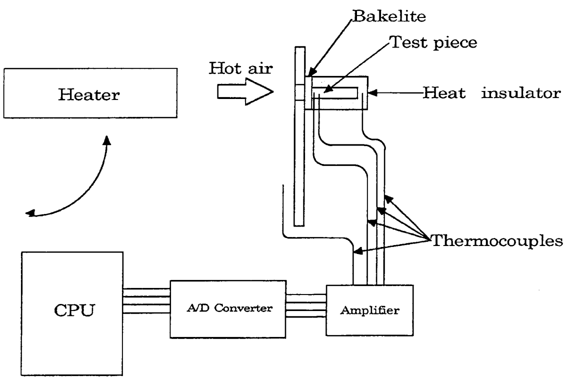

Fig. 5. Schematic of experimental set-up.

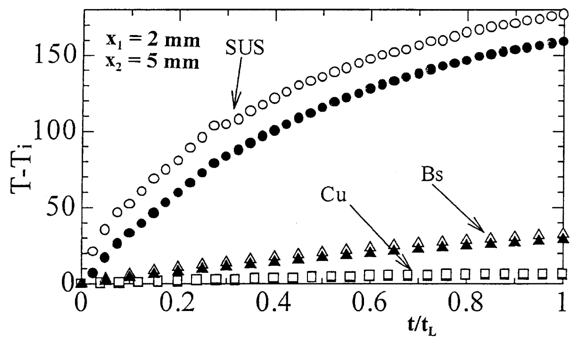

Fig. 6. Temperature change at two points of $x_1 = 2$ and $x_2 = 5$ mm for copper, brass and stainless steel.

($t_L = 6.44$ s for copper and $t_L = 140$ s for stainless steel). It also shows that the temperature for stainless steel rises largely during the measurement, while the rise in the temperature for copper is relatively smaller than that of stainless steel. The rises in the temperature are $T_L - T_i = 180$ K and $T_L - T_i = 6$ K for stainless steel and copper, respectively. During the temperature measurement, the temperature increase for copper becomes about 30 times smaller than stainless steel. This difference largely influences the accuracy of the measurement result when the temperature is measured by equipment with the same minimum division.

5.3. Procedure to determine the initial time

In the numerical case, the initial time is automatically given due to the usage of direct solution. In the actual measurement, however, the initial time is hardly given because it takes time for the thermocouple to sense the temperature change beyond its minimum sensitivity. Therefore, the initial time is defined in the following way as shown in Fig. 7:

1. The time lag, t^* at the point of x_1 can be calculated from the relation of $\min\{(T - T_i)/(T_L - T_i)\} = \mathrm{erfc}(x_1/2\sqrt{at^*}) = 0.01$ and then the temperature, $T^*(x_1, t^*)$ is also given.
2. The point, A, can be plotted as a point of (t^*, T^*) on the t–T graph.
3. The temperature, T_1, first detected by the equipment can be also marked as point, B, on the t–T graph.

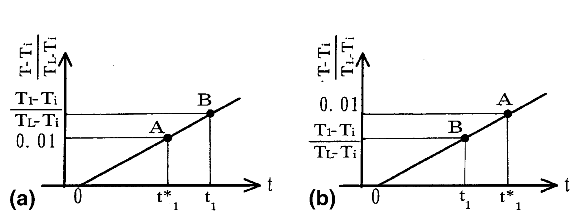

Fig. 7. Procedure to estimate the initial time.

4. As shown in Fig 7, there are two cases (a) or (b) depending on whether T^* is smaller than T_1 or not.
5. The point at which the straight line connecting the points A and B intersects the initial temperature of $T = T_i$ can be determined as the initial time of the measurement.

It may be of importance to say that the difference between the cases (a) and (b) causes little influence on the measured result, although the initial time for case (b) may be more accurate than that of the case (a). After arriving at the initial time, one can calculate the thermal diffusivity by following the explanation in Section 3.2.

5.4. Discussion of measurement result

Fig. 8 shows the values of thermal diffusivity calculated from the temperature change measured for copper, brass and stainless steel. A value of Δx_1 and Δx_2 in Fig. 8 shows uncertainty in units of mm at the points of x_1 and x_2, respectively. It is found from Fig. 8 that for both brass and stainless steel, both values of the thermal diffusivity calculated from the measured values agree with the exact ones and then the level of accuracy is similar to that in the measurement of the temperature. On the other hand, for copper, the error in the estimation becomes relatively larger than that for brass and stainless steel. This results from the error in the temperature measurement. The minimum setting on the measuring equipment corresponds to 0.06 K, while the ratio of the error contained in the measured temperature to the maximum temperature rise becomes $\{0.06/(T_L - T_i)\} \fallingdotseq 0.01$ for copper compared with $\{0.06/(T_L - T_i)\} \fallingdotseq 0.002$ for brass and 0.0005 for stainless steel. In other words, the temperature has been measured in a range slightly larger than the minimum division of the measuring equipment, resulting into a large uncertainty included in the measured values. If the amount of heat is increased for the copper to cause a large rise in the temperature, the calculated value would be improved the same as those for brass and stainless steel. Another way is to employ more accurate equipment which can follow a small temperature change with the corresponding accuracy. Therefore, this matter would be cleared by an improvement of heating or measuring equipment.

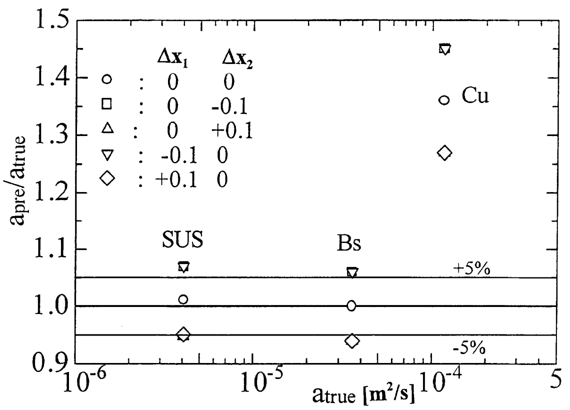

Fig. 8. Accuracy of estimated value and effect of position of measurement on its accuracy.

5.5. Effect of uncertainty included in measuring point on estimated thermal diffusivity

When uncertainties of Δx_1 and $\Delta x_2 = \pm 0.1$ mm are separately added to the true distances of x_1 and x_2, respectively, the influence of these uncertainties on the estimated values is shown in Fig. 8. It is found from Fig. 8 that the uncertainties of ± 0.1 mm overlapped in position, x_1, corresponding to a relative error of 5%, make the estimated value deteriorate by about 5%. Therefore, it may be necessary to give the positions x_1 and x_2 the similar accuracy as the temperature measurement.

5.6. Comparison of the present and existing results

The value of thermal diffusivity can be estimated for brass and stainless steel by the present method as shown in Fig. 8. The same predictive accuracy of about 5% obtained by the experiment can be obtained by the simulation in the previous section. In addition, this accuracy is found to compare to one obtained in the existing procedures, although the present procedure is much simpler than any existing one.

5.7. Merit and caution in the present method

The present method can be applied within the time of t_L, since the inverse solution is employed with the semi-infinite body. The present method has much merit to be totally independent of surface conditions which strongly influence the accuracy of the value estimated by the methods using a direct solution and also to be simpler than the other methods using a direct solution. The number of sampling points is from 30 to 50 well within a measuring time. It may be of importance to clarify that the rise in the temperature measured becomes rapidly larger than the minimum temperature setting of the measuring equipment within a measured time. An improvement of predictive accuracy would be attained up to a level of three significant digits by increasing the order of approximate equation from $N = 7$–9, although the terms of different order of a in $\mathrm{d}F(a)/\mathrm{d}a$ are rapidly increased with increasing N and its functional form becomes complicated. An increase in the order of N in Eq. (4) does not improve the accuracy, since the inverse solution itself cannot be improved by increasing the order more than that of $N = 10$.

Finally, it should be mentioned that in this method, relative temperatures at two points are important in place of absolute ones.

6. Comparison of arbitrary heating method with present method

Although most of the existing methods employed the direct solution, Iida and Shigeta [9,10] developed a different one that is similar to the present one from the viewpoint that both methods do not need a direct solution and are free from a surface condition and employ Laplace transformation. Both employ the same subsidiary Eq. (3). In Eq. (3), Iida and Shigeta [9,10] directly and numerically estimated the thermal diffusivity using the solution in a subsidiary form, which was much different from the present one. In their calculation, the ranges of the maximum measuring time and the value of Laplace operator, s, are experimentally given as $8 \leqslant st_{\max} \leqslant 12$, resulting in a good estimation. The reason that these ranges are enough to predict a good estimate cannot be explained well yet. Therefore, the effect of the value of $t_{\max}$ on the estimated value may be difficult to be predicted, in comparison with the present one in which the maximum time can be exactly given.

7. Estimation of thermal conductivity

Provided that the density for a solid is known, either its thermal conductivity or its heat capacity can be properly estimated, since the thermal diffusivity is already known. However, both properties cannot be estimated from the measurement of the temperature only as shown in Fig. 5, because their dimension includes a unit of heat. In order to estimate one of them, one has to measure one of the two values of heat transferred into the solid and temperature distribution or gradient in the solid beside the temperature change at a given point.

According to the inverse solution, the surface heat flux for the semi-infinite solid is given by the following equation and its accuracy in predicting the surface heat flux corresponds to that in the temperature measurement:

$$\Phi_{\mathrm{w}}(\tau) = \sum_{j=-1}^{N} G'_j(\tau - \tau_1^*)^{j/2} \Big/ \Gamma\left(\frac{j}{2}+1\right), \tag{11}$$

where the details of the coefficients are given in [12].

If the surface heat flux would be measured with the same accuracy as the temperature measurement, the thermal conductivity can be determined from Eq. (11) by using the temperature change given by Eq. (4) as

$$\frac{\lambda}{\sqrt{a}} = \frac{\int_{t_1}^{t_2} q(t)\,\mathrm{d}t}{\int_{t_1}^{t_2} \sum_{j=-1}^{N} G_j(t - t_1^*)^{j/2}/\Gamma(j/2+1)\,\mathrm{d}t}, \tag{12}$$

where

$$G_{-1} = \sum_{k=0}^{Nk} b_k e_k, \quad j = -1, \; Nk = N,$$

$$G_j = \sum_{k=0}^{Nk} b_{k+j+1} e_k, \quad j \geqslant 0, \; Nk = N - j.$$

For the case of a constant heat flux, Eq. (12) can be simplified as

$$\frac{\lambda}{\sqrt{a}} = \frac{q_0(t_2 - t_1)}{\sum_{j=-1}^{N} G_j((t_2 - t_1^*)^{j/2+1} - (t_1 - t_1^*)^{j/2+1})/(\Gamma(j/2+2))}. \tag{13}$$

In the case where the heat flux is still a function of time, one has to use a finite solid as a reference, for which physical properties needed are well known, and to estimate the heat flux from the temperature change in the finite referred solid. Substituting the estimated heat flux into Eq. (12), one can estimate the thermal conductivity. As the result, the finite referred solid makes a simultaneous measurement of thermal diffusivity and thermal conductivity possible.

Although the simultaneous measurement is not made in the present experimental system, a numerical estimate is still possible. We estimate the thermal diffusivity using the temperature change in a solid, as an example, for the constant heat flux indicated by the case (3). For copper, the estimated value of λ is found to become $\lambda = 386.7$ W/m K for an actual value of $\lambda = 386$ W/m K. It is numerically verified that the value of the estimated thermal conductivity also agrees with the actual value for any solid within an accuracy of uncertainty in the measurement, since Eq.(12) is derived from the inverse solution in non-dimensional form.

8. Conclusions

A new method to measure the thermal diffusivity is proposed using the inverse solution for one-dimensional heat conduction and the following results are obtained.

1. The validity of the method proposed for thermal diffusivity is numerically and experimentally verified and then a simultaneous measurement of the thermal diffusivity and thermal conductivity is numerically verified.
2. The thermal diffusivity can be estimated within an accuracy of 2–3% using temperature changes measured at two different positions with an accuracy of 1%.
3. The level of accuracy of the estimated value almost corresponds to that of the inverse solution.
4. The present method has merit in that it is independent of surface condition and simpler than the existing ones.

5. The sampling number needed in the estimation can be determined.

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